Introduction to sequences and series

Approximate

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

Notational Note $n! = n(n-1)(n-2)\cdots(2)(1)$

Use these series to find approx

$\sin\left(\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{6}\right)$

$\sin(\pi)$

$\cos(\pi)$

$Y_1 = 1 - x^2/2! + x^4/4! \cdots$

2nd Quit

[function]

VARS ⇒ ENTER

$Y_1(\pi/6)$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \approx .866025\ldots$$

$$\sin(\pi) = 0$$

$$\cos(\pi) = -1$$

Q: How do we get this A: wait...

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$$

$$\frac{d}{dx}(\cos(x)) \approx -\frac{2x}{2!} + \frac{4x^3}{4!} - \frac{6x^5}{6!} + \frac{8x^7}{8!} - \frac{10x^9}{10!}$$

$$\Downarrow$$

$$-\sin(x) = -\frac{x}{1!} + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \frac{x^9}{9!}$$

$$\sin(x) \approx \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \ldots$$

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$$\cos(x) = \sum_{K=0}^{\infty} (-1)^K \frac{x^K}{K!}$$

NOTATIONAL CONVENTION

$0! = 1$

Historical Note

Graph $x!$ . . . . . nothing happens

EULER

$\Gamma(x)$

$\Gamma(n) = n!$

Sequences

A sequence is an ordered list of thingies. [[numbers for our purposes]]

1, 3, 5, 7

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^{n-1}}, \ldots$

$\left\{\frac{1}{2^{n-1}}\right\}_{n=1}^{\infty}$ like $\frac{1}{2^x}$

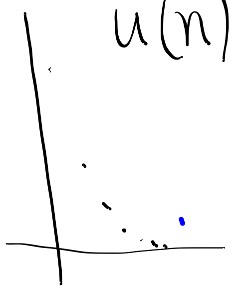
Encyclopedia of Sequences

~~Neal~~

N.J.A. Sloane

nMin=1

$u(n)=1/2^{(n-1)}$



Window

nMin=1

nMax=20

Xmin=-1

Xmax=20

Xscl=.01

Ymin=0

Ymax=1

Yscl=.01

$\lim_{n\to\infty} \{a_n\}$

the n^{th} term of sequence a

Infinite sequences

Decreasing seq: if $n_1 > n_2$
Inc
then $a_{n_1} \leq a_{n_2}$ (≥)

strictly decreasing seq: if $n_1 > n_2$
Inc
then $a_{n_1} < a_{n_2}$ (>)

Monotonic: either decreasing or increasing

$-1, -2, -3, -4, -5, -6, \ldots, -n, \ldots$

strictly decreasing

$$\{-n\}_{n=-4}^{\infty}$$

n: always a non-negative integer unless I say otherwise

A seq is bounded above if there exists an M with $a_n \leq M$ for every n [for every term in seq]

10.2/ 1-10

A seq is bounded below if there is an N with $a_n \geq N$ for ev...

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8.2/39)

$$\bigstar \int_1^3 \sqrt{x}\,\tan^{-1}\sqrt{x}\,dx$$

Let $u = \tan^{-1}\sqrt{x}$ $\quad dv = \sqrt{x}\,dx = x^{1/2}dx$

$$du = \frac{1}{1+(\sqrt{x})^2}\cdot\frac{d}{dx}\left(x^{1/2}\right)dx \qquad v = \frac{2}{3}x^{3/2}$$

$$du = \frac{1}{1+x}\,\frac{1}{2\sqrt{x}}\,dx \qquad v = \frac{2}{3}x^{3/2}$$

$$\bigstar = \frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x} - \frac{1}{3}\int x\,\frac{1}{1+x}\,dx$$

$$= \frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x} - \frac{1}{3}\int\frac{x}{1+x}\,dx$$

$$x+1\overline{)\,x\,}\quad 1\ R\text{-}1 \qquad -(x+1) \qquad -1$$

$$-\frac{1}{3}\int 1 + \frac{-1}{1+x}\,dx$$

$$\bigstar = \frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x} - \frac{1}{3}x + \frac{1}{3}\ln|1+x| + C$$

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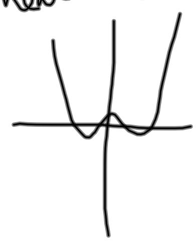
Introduction to sequences and series

[only "good" for xs "near" 0]



Approximate

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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

Use these series to find approx

$\sin\left(\frac{\pi}{6}\right)$ =1/2

$\cos\left(\frac{\pi}{6}\right)$ =.866025...

$\sin(\pi)$ =0

$\cos(\pi)$ =-1

Notational Note $n! = n(n-1)(n-2)\cdots(2)(1)$

(MATH => PRB (enter) => option 4)

$Y_1 = 1 - x^2/2! + x^4/4! \ldots$

2nd Quit

[function]

VARS ⇒ ENTER

$Y_1(\pi/6)$

Goal: Infinite Polynomials that approximate "nice" f^ns

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \ldots$$

$$-\sin x \approx -\frac{2x}{2!} + \frac{4x^3}{4!} - \frac{6x^5}{6!} + \frac{8x^7}{8!} - \frac{10x^9}{10!} + \ldots$$

$$-\sin x \approx -\frac{x}{1!} + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \frac{x^9}{9!} + \ldots$$

$$\sin x \approx \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \ldots$$

good for an interval of convergence

Feb 28-11:47 AM

Shelby asks: how do we get these?

Ans: wait - - - -

Start at sequences....

A sequence is an ordered list of thingies..
(for us these will be numbers)

$1, 3, 5, 7$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots, \frac{1}{2^{n-1}}, \dots$

shelby, brittany, carol, ...

Encyclopedia of Sequences

$$\left\{\frac{1}{2^{n-1}}\right\}_{n=1}^{\infty}$$

f: non-negative integers $\to$ #s

graph

MODE $\to$ SEQ

$Y=$

nMin = 1

$u(n) = 1/2^{(n-1)}$

WINDOW
nMin = 1
nMax = 20
⋮
Xmin = 0
Xmax = 20
Ymin = 0
Ymax = 1
Yscl = .01

MODE DOT
will just show you the pts of the sequence

$$\lim_{n\to\infty}\left\{\frac{1}{2^{n-1}}\right\}=0$$

Sequential jargon

A decreasing sequence $\{a_n\}$

if $n_1 > n_2$

then $a_{n_1} \leq a_{n_2}$

A strictly decreasing sequence

if $n_1 > n_2$ then $a_{n_1} < a_{n_2}$

2

An increasing seq. if $n_1 > n_2$ then $a_{n_1} \geq a_{n_2}$

A strictly increasing seq: if $n_1 > n_2$ then $a_{n_1} > a_{n_2}$

2

A monotonic sequence is an increasing or decreasing seq.

"strictly monotonic"

Feb 28-12:20 PM

$-1, -2, -3, -4, -5, \ldots, -n, \ldots$

A sequence $\{a_n\}$ is bounded above
if there is an M
with $a_n \leq M$ for every n,

A seq $\{a_n\}$ is bounded below
if there is an N
with $a_n \geq N$ for every n

A seq is BOUNDED
if it is bounded ABOVE & bounded BELOW

$-1, +1, -2, +2, -3, +3, \ldots, -$[illegible]$, +$[illegible]$, \ldots$

Every bounded monotonic sequence has a limit